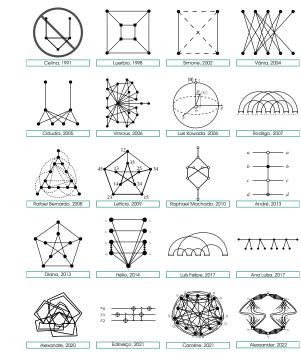
A Perfect Path from Computational Biology to Quantum Computing

Celina Miraglia Herrera de Figueiredo

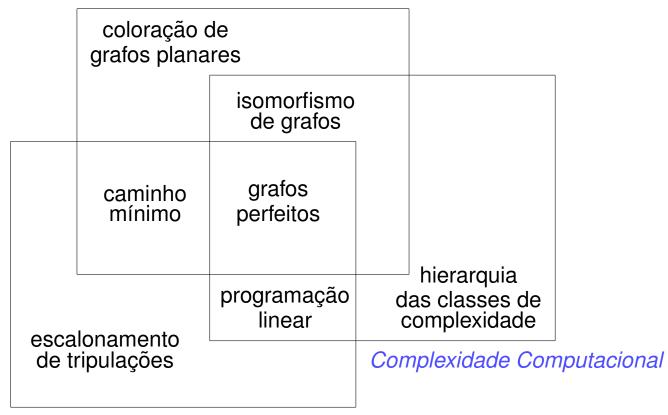






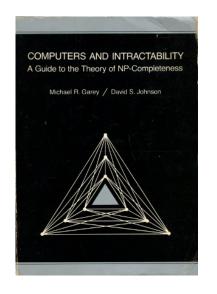
Origem e desenvolvimento da área de pesquisa

Teoria dos Grafos



Otimização Combinatória

The Guide – Computers and Intractability



"Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson."

Lance Fortnow, "Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness"

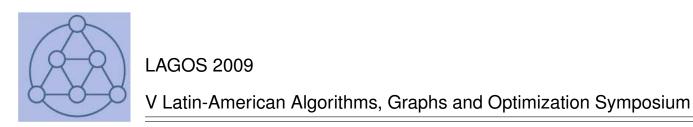
Ongoing Guide - Graph Restrictions and Their Effect

											_											
GRAPH CLASS	ME	MBER	INI	SET	CLIC	QUE	CLI	Par	Сн	RNUM	Сня	IND	HAN	иCIR	Do	иSет	MA	хСит	STT	REE	GRA	Iso
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Almost Trees (k)	P		P	[24]	P	[T]	P?		P?		P?		P?		P	[45]	P?		P?		P?	
Partial k-Trees	P	[2]	P	[1]	P	[T]	P?		P	[1]	0?		P	[3]	P	[3]	P?		P?		0?	
Bandwidth-k	P	[68]	P	[64]	P	[T]	P?		P	[64]	P?		P?		P	[64]	P	[64]	P?		P	[58]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]	N	[49]	N	[GJ]	N	[GJ]	N	[GJ]	N	[GJ]	P	[58]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	o		N	[GJ]	N	[GJ]	P	[GJ]	N	[35]	P	[GJ]
Series Parallel	P	[79]	P	[75]	P	[T]	P?		P	[74]	P	[74]	P	[74]	P	[54]	P	[GJ]	P	[82]	P	[GJ]
Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[67]	P	[67]	P	[T]	P	[6]	P	[GJ]	P	[81]	P	[GJ]
Halin	P		P	[6]	P	[T]	P	[6]	P	[74]	P	[74]	P	[T]	P	[6]	P	[GJ]	P?		P	[GJ]
k-Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[6]	O?		P	[6]	P	[6]	P	[GJ]	P?		P	[GJ]
Grid	P		P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[51]	N	[55]	P	[T]	N	[35]	P	[GJ]
$K_{3,3}$ -Free	P	[4]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	0?		N	[GJ]	N	[GJ]	P	[5]	N	[GJ]	O ?	
Thickness-k	N	[60]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	N	[49]	N	[GJ]	N	[GJ]	N	[7]	N	[GJ]	\mathbf{O} ?	
Genus-k	P	[34]	N	[GJ]	P	[T]	N	[10]	N	[GJ]	O?		N	[GJ]	N	[GJ]	O?		N	[GJ]	P	[61]
Perfect	O!		P	[42]	P	[42]	P	[42]	P	[42]	0?		N	[1]	N	[14]	0?		N	[GJ]	I	[GJ]
Chordal	P	[76]	P	[40]	P	[40]	P	[40]	P	[40]	0?		N	[22]	N	[14]	0?		N	[83]	I	[GJ]
Split	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	0?		N	[22]	N	[19]	0?		N	[83]	I	[15]
Strongly Chordal	P	[31]	P	[40]	P	[40]	P	[40]	P	[40]	O?		0?		P	[32]	0?		P	[83]	O?	
Comparability	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	O?		N	[1]	N	[28]	0?		N	[GJ]	I	[GJ]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[1]	N	[28]	P	[T]	N	[GJ]	I	[GJ]
Permutation	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]	O?		O		P	[33]	O?		P	[23]	P	[21]
Cographs	P	[T]	P	[40]	P	[40]	P	[40]	P	[40]	O?		P	[25]	P	[33]	O ?		P	[23]	P	[25]
Undirected Path	P	[39]	P	[40]	P	[40]	P	[40]	P	[40]	0?		0?		N	[16]	0?		0?		I	[GJ]
Directed Path	P	[38]	P	[40]	P	[40]	P	[40]	P	[40]	0?		0?		P	[16]	0?		P	[83]	O?	
Interval	P	[17]	P	[44]	P	[44]	P	[44]	P	[44]	0?		P	[53]	P	[16]	0?		P	[83]	P	[57]
Circular Arc	P	[78]	P	[44]	P	[50]	P	[44]	N	[36]	0?		0?		P	[13]	0?		P	[83]	0?	
Circle	P	[71]	P	[GJ]	P	[50]	0?		N	[36]	0?		P	[12]	0?		0?		P	[70]	0?	
Proper Circ. Arc	P	[77]	P	[44]	P	[50]	P	[44]	P	[66]	0?		P	[12]	P	[13]	0?		P	[83]	0?	
Edge (or Line)	P	[47]	P	[GJ]	P	[T]	N	[GJ]	N	[49]	0?		N	[11]	N	[GJ]	0?		N	[70]	I	[15]
Claw-Free	P	[T]	P	[63]	O?		N	[GJ]	N	[49]	O?		N	[11]	N	[GJ]	O ?		N	[70]	I	[15]

The updated NP-Completeness Column: An Ongoing Guide table 35 years later

GRAPH CLASS	ME	MBER	INI	SET	CL	IQUE	CL	PAR	Сн	RNUM	CHR	Ind	HA	MCIR	Do	MSET	MAX	CUT	ST	ΓREE	GR	APHISC
TREES/FORESTS	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[T]	Р	[GJ]	Р	[GJ]	Р	[T]	Р	[GJ]
ALMOST TREES (k)	Р	[OG]	Р	[OG]	Р	[T]	P	[105]	P	[5]	P	[17]	Р	[5]	Р	[5]	P	[20]	P	[76]	P	[17]
PARTIAL K-TREES	Р	[OG]	Р	[5]	Р	[T]	P	[105]	Р	[5]	P	[17]	Р	[5]	Р	[5]	P	[20]	P	[76]	P	[17]
BANDWIDTH-K	Р	[OG]	Р	[OG]	Р	[T]	P	[105]	Р	[5]	P	[17]	Р	[5]	Р	[5]	P	[OG]	P	[76]	Р	[OG]
Degree-k	Р	[T]	N	[GJ]	Р	[T]	N	[29]	Ν	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[GJ]	Ν	[GJ]	Р	[OG]
PLANAR	Р	[GJ]	N	[GJ]	Р	[T]	N	[78]	N	[GJ]	0		N	[GJ]	N	[GJ]	Р	[GJ]	N	[OG]	Р	[GJ]
SERIES PARALLEL	Р	[OG]	Р	[OG]	Р	[T]	P	[105]	Р	[5]	P	[17]	Р	[5]	Р	[OG]	P	[GJ]	Р	[OG]	Р	[GJ]
OUTERPLANAR	Р	[OG]	Р	[OG]	Р	[T]	P	[OG]	Р	[OG]	P	[OG]	Р	[T]	Р	[OG]	P	[GJ]	Р	[OG]	Р	[GJ]
HALIN	Р	[OG]	Р	[OG]	Р	[T]	P	[OG]	Р	[5]	P	[17]	Р	[T]	Р	[OG]	P	[GJ]	P	[118]	Р	[GJ]
k-Outerplanar	Р	[OG]	Р	[OG]	Р	[T]	Р	[OG]	Р	[5]	P	[17]	Р	[OG]	Р	[OG]	Р	[GJ]	Р	[76]	Р	[GJ]
GRID	Р	[OG]	Р	[GJ]	Р	[T]	P	[GJ]	Р	[T]	P	[GJ]	N	[OG]	N	[32]	P	[T]	N	[OG]	Р	[GJ]
K _{3,3} -Free	Р	[OG]	N	[GJ]	Р	[T]	N	[78]	N	[GJ]	0?		N	[GJ]	N	[GJ]	P	[OG]	N	[GJ]	P	[40]
THICKNESS-K	N	[OG]	N	[GJ]	Р	[T]	N	[78]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[119]	N	[GJ]	1	[RJ]
GENUS-k	Р	[OG]	N	[GJ]	Р	[T]	N	[78]	Ν	[GJ]	0?		N	[GJ]	N	[GJ]	0?		Ν	[GJ]	Р	[OG]
PERFECT	P	[34]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	N	[28]	N	[OG]	N	[OG]	N	[20]	N	[GJ]	1	[84]
CHORDAL	Р	[OG]	Р	[OG]	Р	[OG]	P	[OG]	P	[OG]	0?		N	[93]	N	[OG]	N	[20]	N	[OG]	1	[84]
SPLIT	Р	[OG]	Р	[OG]	Р	[OG]	P	[OG]	P	[OG]	0?		N	[93]	N	[OG]	N	[20]	N	[OG]	1	[108]
STRONGLY CHORDAL	Р	[OG]	Р	[OG]	Р	[OG]	P	[OG]	P	[OG]	0?		N	[93]	P	[OG]	N	[109]	P	[OG]	1	[111]
Comparability	Р	[OG]	Р	[OG]	Р	[OG]	P	[OG]	P	[OG]	N	[28]	N	[OG]	N	[94]	N	[102]	N	[GJ]	-1	[22]
BIPARTITE	Р	[T]	Р	[GJ]	Р	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[OG]	N	[94]	P	[T]	N	[GJ]	-1	[22]
PERMUTATION	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]			Р	[44]	Р	[OG]	N	[120]	Р	[OG]	Р	[OG]
Cographs	Р	[T]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	0?		Р	[OG]	Р	[OG]	Р	[20]	Р	[OG]	Р	[OG]
UNDIRECTED Path	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	0?		N	[13]	N	[OG]	N	[20]	N	[RJ]	1	[22]
DIRECTED PATH	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	0?		N	[99]	Р	[OG]	N	[1]	Р	[OG]	P	[7]
INTERVAL	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	0?		Р	[OG]	Р	[OG]	N	[1]	Р	[OG]	Р	[OG]
CIRCULAR ARC	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	N	[OG]	0?		Р	[106]	Р	[OG]	N	[1]	Р	[11]	Р	[80]
CIRCLE	Р	[OG]	Р	[GJ]	Р	[OG]	N	[73]	N	[OG]	0?		N	[39]	N	[71]	N	[26]	Р	[<u>OG</u>]	Р	[68]
PROPER CIRC. ARC	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	Р	[OG]	0?		Р	[OG]	Р	[OG]	0?		Р	[11]	Р	[82]
EDGE (OR LINE)	Р	[OG]	Р	[GJ]	Р	[T]	N	[95]	N	[OG]	N	[28]	N	[OG]	N	[GJ]	Р	[59]	N	[19]	1	[OG]
CLAW-FREE	Р	[T]	Р	[OG]	N	[103]	N	[85]	N	[OG]	N	[28]	N	[OG]	N	[GJ]	N	[20]	N	[19]	1	[OG]

www.cos.ufrj.br/~celina/ftp/j/RJ-current.pdf





The P vs. NP-complete dichotomy of some challenging problems in graph theory

Celina de Figueiredo

Universidade Federal do Rio de Janeiro Brazil

November 2009

Two long-standing problems in graph theory

Perfect graphs: Chvátal's SKEW PARTITION is polynomial

Intersection graphs: Roberts—Spencer's CLIQUE GRAPH is NP-complete

Both SKEW PARTITION and CLIQUE GRAPH proved to be in NP when their classification into P or NP-complete was proposed

V. Chvátal – J. Combin. Theory Ser. B 1985

F. Roberts, J. Spencer – J. Combin. Theory Ser. B 1971

The three nonempty part problem

Full dichotomy for the RECOGNITION PROBLEM:
STABLE CUTSET, 3-COLORING are the only NP-complete

T. Feder, P. Hell, S. Klein, R. Motwani – SIAM J. Discrete Math. 2003

Full dichotomy for the SANDWICH PROBLEM:
61 interesting problems: 19 NP-complete, 42 polynomial

HOMOGENEOUS SET SANDWICH PROBLEM is polynomial CLIQUE CUTSET SANDWICH PROBLEM is NP-complete Full dichotomy for the GENERALIZED SPLIT GRAPH SANDWICH PROBLEM: (2,1)-GRAPH SANDWICH PROBLEM is NP-complete

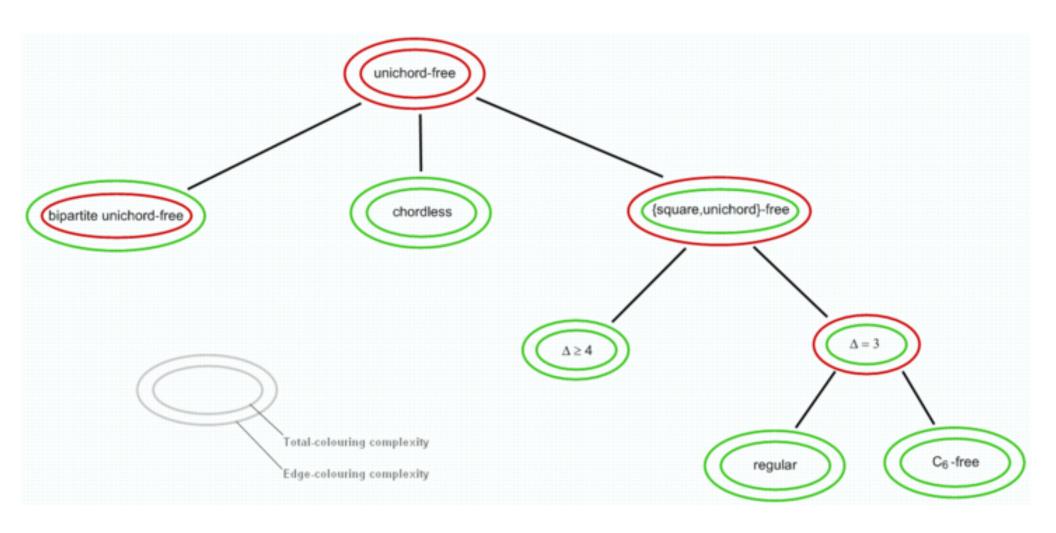
"The polynomial dichotomy for three nonempty part sandwich problems" Discrete Appl. Math. 2009 (with Rafael Teixeira, Simone Dantas) Complexity-separating graph classes for vertex, edge and total coloring



Celina de Figueiredo



Edge and total coloring complexity-separating classes



When restricted to {square,unichord}-free graphs, edge coloring is NP-complete whereas total coloring is polynomial

Complexity restricted to unichord-free and special subclasses

Colouring problem \ class	General	Unichord-free	$\{\Box, \text{unichord}\}$ -free	$\{\triangle, unichord\}$ -free
Vertex-col.	NPC [14]	P [26]	P [26]	P [26]
Edge-col.	\mathcal{NPC} [13]	\mathcal{NPC} [18]	\mathcal{NPC} [18]	\mathcal{NPC} [18]
Total-col.	\mathcal{NPC} [21]	\mathcal{NPC} [17]	P[16,17]	\mathcal{NPC} [17]
Clique-col.	$\Sigma_2^p \mathcal{C}$ [20]	${\cal P}$	${\cal P}$	$\mathcal{P}\left(\kappa=\chi\right)$
Biclique-col.	$\Sigma_2^p \mathcal{C}$ [10]	${\cal P}$	${\cal P}$	$\mathcal{P}\left(\kappa_{B}=2\right)$

^[10] M. Groshaus, F. Soulignac, P. Terlisky – J. Graph Algorithms Appl. 2014

"Efficient algorithms for clique-colouring and biclique-colouring unichord-free graphs" *Algorithmica* 2017 (with Hélio Macedo and Raphael Machado)

^[20] D. Marx - Theoret. Comput. Sci. 2011

Dániel Marx plenary talk at ICGT 2014

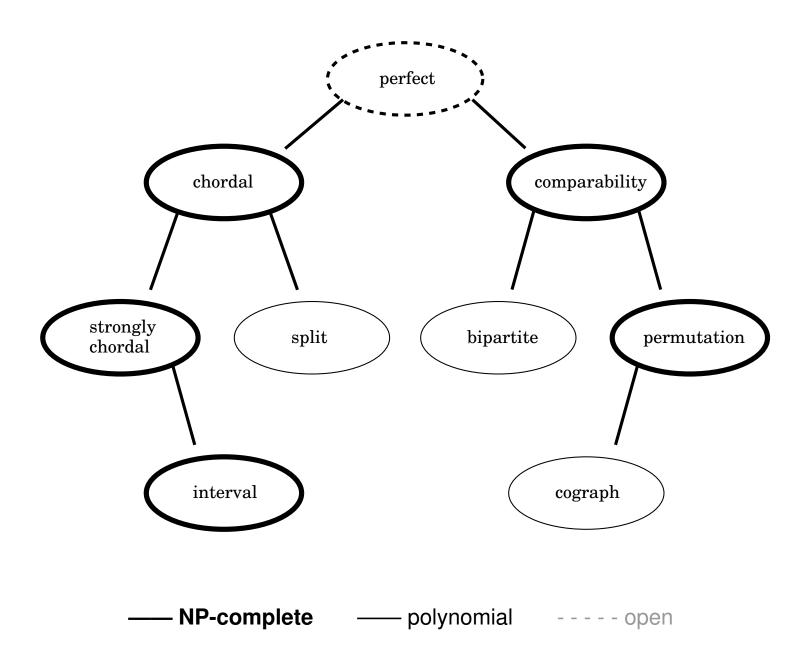
Every graph is easy or hard: dichotomy theorems for graph problems

Dániel Marx¹

¹Institute for Computer Science and Control, Hungarian Academy of Sciences (MTA SZTAKI) Budapest, Hungary

> ICGT 2014 Grenoble, France July 3, 2014

Sandwich problems for perfect graph classes



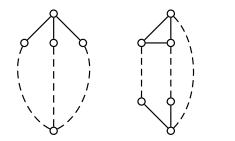
Is the not C-free easier than the C-free sandwich problem?

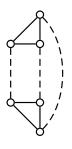
A trigraph (G_1,G_2) satisfies property Π if there is no sandwich graph G for (G_1,G_2) which does not satisfy Π

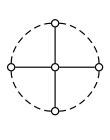
The recognition of Berge graphs is polynomial but the recognition of Berge trigraphs was previously open

The imperfect graph sandwich problem is polynomial Equivalently, recognizing Berge trigraphs is polynomial

Detecting 3-path configurations







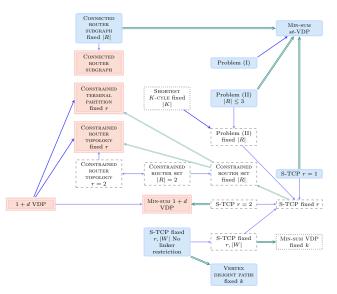
theta and pyramid: polynomial

prism and wheel: NP-complete

The not pyramid-free sandwich problem is polynomial but the complexity of the pyramid-free sandwich problem is open

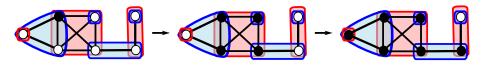
"The world of hereditary graph classes viewed through Truemper configurations" by K. Vušković, in *Surveys in Combinatorics* (2013)

Vertex-disjoint paths and connection problems



"On undirected two-commodity integral flow, disjoint paths and strict terminal connection problems", *Networks* (2021) (with Alexsander Melo, Uéverton Souza)

A quantum walker spreads across a 2-tessellation cover



The chromatic upper bound: $T(G) \leq \min\{\chi'(G), \chi(K(G))\}\$

"The graph tessellation cover number: Chromatic bounds, efficient algorithms and hardness" Theoretical Computer Science (2020) (with Alexandre Abreu, Luis Cunha, Luis Kowada, Franklin Marquezino, Daniel Posner, Renato Portugal)

Most significant publications

- FIGUEIREDO, C. M. H. · KLEIN, S. · KOHAYAKAWA, Y. · REED, B.

 Finding skew partitions efficiently

 Journal of Algorithms (2000)
- FIGUEIREDO, C. M. H. · MAFFRAY, F.

 Optimizing bull-free perfect graphs

 SIAM Journal on Discrete Mathematics (2004)
- FARIA, L. · FIGUEIREDO, C. M. H. · SYKORA, O. · VRTO, I.

 An improved upper bound on the crossing number of the hypercube

 Journal of Graph Theory (2008)
- ALCON, L. · FARIA, L. · FIGUEIREDO, C. M. H. · GUTIERREZ, M. The complexity of clique graph recognition
 Theoretical Computer Science (2009)
- FIGUEIREDO, C. M. H.

 The P vs. NP-complete dichotomy of some challenging problems in graph theory
 Discrete Applied Mathematics (2012)

Most significant publications

- CUNHA, L. F. I. · KOWADA, L. A. B. · HAUSEN, R. A. · FIGUEIREDO, C. M. H.

 A faster 1.375-approximation algorithm for sorting by transpositions

 Journal of Computational Biology (2015)
- MACÊDO, H. B. · MACHADO, R. C. S. · FIGUEIREDO, C. M. H. Hierarchical complexity of 2-clique-colouring weakly chordal graphs and perfect graphs having cliques of size at least 3 Theoretical Computer Science (2016)
- CHUDNOVSKY, M. · FIGUEIREDO, C. M. H. · SPIRKL, S.

 The sandwich problem for decompositions and almost monotone properties

 Algorithmica (2018)
- MELO, A. A. · FIGUEIREDO, C. M. H. · SOUZA, U. S.
 A multivariate analysis of the strict terminal connection problem
 Journal of Computer and System Sciences (2020)
- ABREU, A. · CUNHA, L. · FIGUEIREDO, C. · KOWADA, L. · MARQUEZINO, F. · POSNER, D. · PORTUGAL, R. The graph tessellation cover number: Chromatic bounds, efficient algorithms and hardness Theoretical Computer Science (2020)